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The views expressed in this paper are those of the author.
No responsibility for them should be attributed to the Bank of Canada.

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Abstract

This paper develops a dynamic, stochastic, general-equilibrium (DGSE) model for the Canadian economy and evaluates the real effects of monetary policy shocks. To generate high and persistent real effects, the model combines nominal frictions in the form of costly price adjustment with real rigidities modelled as convex costs of adjusting capital and employment. The structural parameters identifying transmission channels are estimated econometrically using a maximum-likelihood procedure with a Kalman filter. The estimated nominal and real rigidities impart substantial real and persistent effects following a monetary policy shock. Furthermore, the results show that the monetary authority has accommodated technology shocks and has successfully insulated the Canadian economy from demand-side disturbances, by responding to technology and money demand shocks.

JEL classification: E31, E32
Bank classification: Monetary policy framework

Résumé

Dans la présente étude, l'auteur élabore un modèle d'équilibre général dynamique et stochastique (EGDS) de l'économie canadienne. Afin de générer des effets réels considérables et persistants des chocs monétaires, il intègre à ce modèle des frictions nominales et réelles sous la forme de coûts d'ajustement des prix, du capital et de l'emploi. L'auteur estime les paramètres structurels du modèle avec de données canadiennes, en utilisant la méthode du maximum de vraisemblance et le filtre de Kalman. Les versions du modèle d'EGDS doté de rigidités nominales et réelles génèrent des effets réels significatifs et persistants en réaction à des chocs de politique monétaire. De plus, les résultats montrent que l'autorité monétaire réussit bien à absorber les chocs technologiques et à protéger l'économie des perturbations exogènes de la demande de monnaie.

Classification JEL : E31, E32
Classification de la Banque : Cadre de la politique monétaire

1 Introduction

In recent years, there has been a renewed interest in the development of macroeconomic models that emphasize the role of nominal price rigidities (see the synthesis by Goodfriend and King 1997 and the references therein). These models rest on the optimizing behaviour of rational agents in a dynamic, stochastic, general-equilibrium (DSGE) environment. However, Chari, Kehoe, and McGrattan (CKM) (2000) point out that these models, despite the merit of explicitly accounting for the relationship between the behaviour of aggregate quantities and prices, and the decisions of utility-maximizing households and profit-maximizing firms, suffer from a serious anomaly: they generate only weak persistence of real and nominal variables in response to money supply shocks, in contrast to the bulk of evidence indicating that the effects of monetary policy shocks on those variables last several quarters.[1]

This failure of sticky-price models has sparked a rapidly growing literature aimed at identifying alternative transmission channels of monetary policy shocks. Examples include Kiley (1997), who shows that greater persistence arises only if the degree of increasing returns to scale at the individual firm level is large; Gust (1997), who demonstrates that constraining factor mobility across sectors may increase persistence in the presence of staggered price contracts; Huang and Liu (1998), who find that more persistence can be produced under staggered wage contracts than under staggered price contracts; and Bergin and Feenstra (1998), who obtain more persistence if the share in the fixed factor is sufficiently large in a model that features a staggered price mechanism, non-constant elasticity of substitution (CES) production, and factor specificity.

In a static framework of price-setting agents, Ball and Romer (1990) have demonstrated that the degree of nominal rigidity arising from a given menu cost increases with the degree of real rigidity, thus producing larger non-neutralities.[2] Nevertheless, real

[1] More specifically, CKM (2000) develop a one-shock model with imperfect competition in the goods markets and staggered price contracts, in the spirit of Taylor (1980).

[2] The degree of nominal rigidity can be defined as the significance and the duration of nominal shock effects on real variables.

rigidity does not imply nominal rigidity. In other words, without nominal frictions, prices fully adjust in response to money supply shocks regardless of the extent of real rigidity. Thus, money is neutral in the short term.

Real rigidities may arise in the goods and/or labour markets. In the absence of real rigidities, the marginal production cost quickly adjusts in response to money supply shocks, implying non-persistent real effects.[3] The incorporation of capital- and labour-market frictions in a model with costly price adjustments induces a gradual response of real variables to aggregate disturbances. In turn, the marginal production costs of price-setting firms also adjust more slowly. Thus, the combination of nominal and real rigidities can potentially impart a larger nominal price rigidity and a more persistent effect from money supply shocks.

Dib and Phaneuf (2001) argue that Ball and Romer's (1990) original intuition of combining nominal and real rigidities has the potential to substantially increase the real persistence of monetary effects in DSGE models with sticky prices. Dib and Phaneuf (2001) develop and estimate a DSGE model with price- and employment-adjustment costs for the U.S. economy. They find that the impact of nominal rigidity substantially increases in the presence of real rigidity. Following their study, the present paper develops for the Canadian economy an econometric DSGE model with nominal and real rigidities. This model combines nominal frictions in the form of the quadratic costs of changing prices, as in Rotemberg (1982), with real rigidities modelled as the convex costs of adjusting capital and employment, following Sargent (1978). The structural parameters, including the price- and labour-adjustment cost parameters, are estimated using quarterly, seasonally adjusted Canadian data on output, inflation, and money growth from 1976Q1 to 2000Q4. I estimate four versions of the DSGE model: the standard sticky-price model, a model with price and capital rigidities, a model with price and employment rigidities, and a model with all these rigidities. The parameters are estimated using Hansen and Sargent's (1998) procedure of applying a maximum-likelihood method and a Kalman

[3] In standard sticky-price models with flexible capital and labour inputs, the marginal production cost of price-setting firms is a weighted average of the real rental rate on capital and the real wages.

filter to the model's state-space form.

The combination of nominal and real frictions significantly increases the degree of nominal price rigidity. Because the estimated costs of changing prices rise correspondingly, firms, although they face employment-adjustment costs, are reluctant to change their prices in response to changes in aggregate demand.

Performing various simulations based on the estimated models, I find that the models with nominal and real rigidities produce results that differ sharply from those found in standard sticky-price models.[4] The effects of money supply shocks last approximately seven quarters, and money supply shocks contribute substantially to the observed short-run variation in real variables. In addition, adding real frictions to the sticky-price model significantly reduces output and inflation volatility. However, Ellison and Scott (2000) find that standard sticky-price models generate extreme volatility in output and inflation. I also show that arbitrarily increasing the size of price-adjustment costs has no impact on the persistence of real deviations following a money supply shock, unless there are real rigidities. Hence, this result corroborates CKM's (2000) main finding in their sticky-price model.

This paper is organized as follows. Section 2 presents the DSGE model with nominal and real rigidities. Section 3 describes the econometric procedure and discusses the estimation results. In Section 4, I evaluate the implications of the estimated model. Section 5 contains concluding remarks.

2 The Model

This model's basic structure is inspired by Rotemberg (1982), Blanchard and Kiyotaki (1987), Hairault and Portier (1993), Rotemberg (1996), Ireland (1997), and Rotemberg and Woodford (1997). I assume that the economy is populated by a representative household, a representative final-good-producing firm, a continuum of intermediate-good-

[4]With no real rigidities, the estimated costs of changing prices are always quite small. The impulse response of output following a money supply shock dies after one quarter, and the fraction of the total variance of output attributable to money supply shocks is very small even at short horizons.

producing firms indexed by $j \in (0,1)$, and a monetary authority. The representative firm producing the final good sells its output, y_t, on a perfectly competitive market at a price p_t. On the other hand, each intermediate-good-producing firm produces a distinct, perishable, intermediate good, y_{jt}, that it sells on a monopolistically competitive market at a price p_{jt}. The intermediate-good-producing firm pays two distinct finite costs when it adjusts its nominal price and labour input.

2.1 The household

Following Ireland (1997) and Kim (2000), the representative household derives utility from consumption, c_t, real money balances, M_t/p_t, and leisure, $(1-h_t)$. The household's preferences are described by the expected utility function,

$$U_0 = E_0 \sum_{t=0}^{\infty} \beta^t u\left(c_t, \frac{M_t}{p_t}, h_t\right), \tag{1}$$

where $\beta \in (0,1)$ is the discount factor. I assume that the single-period utility function is specified as

$$u(\cdot) = \frac{\gamma}{\gamma - 1} \log\left[c_t^{\frac{\gamma-1}{\gamma}} + b_t^{\frac{1}{\gamma}} \left(\frac{M_t}{p_t}\right)^{\frac{\gamma-1}{\gamma}}\right] + \eta \log\left(1 - h_t\right), \tag{2}$$

where γ and η are positive structural parameters. As in Kim (2000), the preference shock, b_t, can be interpreted as a shock to money demand. This shock follows the autoregressive process:

$$\log(b_t) = (1 - \rho_b)\log(b) + \rho_b \log(b_{t-1}) + \varepsilon_{bt}, \tag{3}$$

where $\rho_b \in (-1,1)$, and the serially uncorrelated shock, ε_{bt}, is normally distributed with mean zero and standard deviation σ_b.

The household enters period t with k_t units of capital and a predetermined M_{t-1} units of money. During period t, the household supplies units of capital and labour to each intermediate-good-producing firm. Hence, its choices of h_t and k_t must satisfy $h_t = \int_0^1 h_{jt} dj$ and $k_t = \int_0^1 k_{jt} dj$ for all $t = 0, 1, 2, \cdots$, where h_t represents total hours

worked. In addition to its capital and labour incomes, the household receives a lump-sum nominal transfer from the central bank, T_t, and dividend payments from intermediate-good-producing firms, $D_t = \int_0^1 D_{jt} dj$. The household uses some of its funds to purchase the final output at the nominal price, p_t, which it then divides between consumption and investment. Investment, i_t, increases the capital stock, k_t, over time according to

$$k_{t+1} = (1 - \delta) k_t + i_t, \tag{4}$$

where $\delta \in (0, 1)$ is a constant capital depreciation rate. Furthermore, I assume that it is costly to intertemporally adjust capital and that the capital adjustment cost is specified as

$$CAC_t = \frac{\phi_k}{2} \frac{i_t^2}{k_t}, \tag{5}$$

where $\phi_k > 0$ is the capital adjustment cost parameter. With this configuration, the cost of changing the capital stock increases with the speed of desired adjustment, giving the household an incentive to change investment gradually.

The household's budget constraint is therefore given by

$$c_t + i_t + \frac{\phi_k}{2} \frac{i_t^2}{k_t} + \frac{M_t}{p_t} \leq r_t k_t + w_t h_t + \frac{M_{t-1} + T_t + D_t}{p_t}, \tag{6}$$

where r_t and w_t denote the real capital rental rate and the real wage, respectively.

Given initial values, the household chooses $\{c_t, M_t, h_t, k_{t+1}, i_t\}$, $t = 0, 1, 2, \cdots$, to maximize in each period the expectation of the discounted sum of its utility flows subject to the capital accumulation equation and the budget constraint. The problem can be written in its recursive form, with the optimal solution satisfying the following Bellman equation

$$V(k_t, M_{t-1}, \Omega_t) = \max_{\{c_t, M_t, h_t, k_{t+1}, i_t\}} \left[u\left(c_t, \frac{M_t}{p_t}, h_t\right) + \beta E_t V(k_{t+1}, M_t, \Omega_{t+1}) \right]$$

with respect to constraints (4) and (6), where Ω_t is the information set upon which expectations formed in period t are conditioned. The first-order conditions for this problem

are:

$$\frac{c_t^{-\frac{1}{\gamma}}}{c_t^{\frac{\gamma-1}{\gamma}} + b_t^{\frac{1}{\gamma}} (M_t/p_t)^{\frac{\gamma-1}{\gamma}}} - \lambda_t = 0; \tag{8}$$

$$\frac{b_t^{\frac{1}{\gamma}} (M_t/p_t)^{-\frac{1}{\gamma}}}{c_t^{\frac{\gamma-1}{\gamma}} + b_t (M_t/p_t)^{\frac{\gamma-1}{\gamma}}} - \lambda_t + \beta E_t \left(\frac{p_t \lambda_{t+1}}{p_{t+1}} \right) = 0; \tag{9}$$

$$\frac{\eta}{1-h_t} - \lambda_t w_t = 0; \tag{10}$$

$$\beta E_t \left[\frac{\lambda_{t+1}}{\lambda_t} \left(r_{t+1} + \frac{\phi_k}{2} \left(\frac{i_{t+1}}{k_{t+1}} \right)^2 + (1-\delta) \left(1 + \phi_k \frac{i_{t+1}}{k_{t+1}} \right) \right) \right]$$

$$-\phi_k \frac{i_t}{k_t} - 1 = 0; \tag{11}$$

$$k_{t+1} - (1-\delta) k_t - i_t = 0; \tag{12}$$

where λ_t is the Lagrangian multiplier associated with the household's budget constraint.

Equations (8) and (10) equate the marginal rate of substitution between consumption and labour to the real wage. Equation (9) stipulates that the marginal utility of real money balances is equal to the difference between the current marginal utility of consumption and the expected future marginal utility of consumption adjusted for the expected inflation rate. Equation (11) corresponds to the optimal intertemporal wealth allocation.

As in Ireland (1997) and Kim (2000), equations (8) and (9) imply the following standard money demand function:

$$\log \left(\frac{M_t}{p_t} \right) \approx \log(c_t) - \gamma \log(R_t) + \log(b_t), \tag{13}$$

where R_t denotes the net nominal interest rate between t and $t+1$, and $-\gamma$ is money-interest elasticity.[5] Thus, b_t represents a serially correlated shock to money demand.

[5] In this model, $R_t = \frac{\lambda_t / p_t}{\beta E_t (\lambda_{t+1}/p_{t+1})} - 1$.

2.2 The final-good-producing firm

The final good is produced from a continuum of intermediate goods. Assuming that all intermediate goods are imperfect substitutes with a CES, θ, the corresponding Dixit-Stiglitz (1977) aggregator can be defined as

$$y_t \leq \left(\int_0^1 y_{jt}^{\frac{\theta-1}{\theta}} dj \right)^{\frac{\theta}{\theta-1}}, \theta > 1. \tag{14}$$

Given the relative price vector, the final-good-producing firm chooses the quantity of intermediate good y_{jt} that maximizes its profits. The optimization problem is

$$\max_{y_{jt}} \left[p_t \left(\int_0^1 y_{jt}^{\frac{\theta-1}{\theta}} dj \right)^{\frac{\theta}{\theta-1}} - \int_o^1 p_{jt} y_{jt} dj \right].$$

The first-order condition implies the following demand function for firm j:

$$y_{jt} = \left(\frac{p_{jt}}{p_t} \right)^{-\theta} y_t, \tag{15}$$

which expresses the demand for good j as a function of its relative price and final output. The final-good price index satisfies

$$p_t = \left(\int_o^1 {p_{jt}}^{1-\theta} dj \right)^{\frac{1}{1-\theta}}. \tag{16}$$

2.3 The intermediate-good-producing firm

Intermediate-good-producing firm j hires k_{jt} units of capital and h_{jt} units of labour to produce output according to the following constant-returns-to-scale technology:

$$y_{jt} \leq A_t k_{jt}^{\alpha} \left(g^t h_{jt} \right)^{1-\alpha}, \quad \alpha \in (0,1) \text{ and } g \geq 1, \tag{17}$$

where g is the growth rate of labour productivity (which is also the growth rate of the economy), and A_t is a technology shock that is common to all intermediate-good-producing firms. The technology shock, A_t, is assumed to follow the autoregressive process

$$\log A_t = (1 - \rho_A) \log(A) + \rho_A \log(A_{t-1}) + \varepsilon_{At}, \tag{18}$$

where $\rho_A \in (-1, 1)$, and ε_{At} is a serially uncorrelated shock that is normally distributed with zero mean and standard deviation σ_A.

It is well-known that money is neutral in a monopolistic competition framework (except for the inflation tax effect) unless some sort of nominal friction is added to the model (e.g., Rotemberg 1982). Here, nominal rigidity is introduced by the presence of price-adjustment costs. I assume that the intermediate-good-producing firm faces a quadratic cost of adjusting its nominal price given by the following function:

$$PAC_{jt} = \frac{\phi_p}{2}\left(\frac{p_{jt}}{p_{jt-1}} - 1\right)^2 y_t, \tag{19}$$

where $\phi_p \geq 0$ is the price-adjustment cost parameter. These real costs are measured in terms of the final good. Rotemberg (1982) interprets this quadratic adjustment cost specification as capturing the negative effects of price changes on consumer-firm relationships, which increase in magnitude with the size of the price change and with the overall scale of economic activity, as summarized by total output of the finished good. The price-markup is constant under complete price flexibility ($\phi_p = 0$), but it is endogenous when prices are rigid.[6]

The second real rigidity is a labour-market friction. Specifically, intermediate firm j pays the convex costs of varying its labour input according to the following adjustment cost function:

$$EAC_{jt} = \frac{\phi_h}{2}\left(\frac{h_{jt}}{h_{jt-1}} - 1\right)^2 y_t, \tag{20}$$

where $\phi_h \geq 0$ is the employment-adjustment cost parameter.[7] These costs are also measured in terms of the final good, and they directly affect labour demand. The cost of adjusting employment in response to aggregate shocks increases with the speed of the desired adjustment. This gives firms an incentive to undertake employment changes

[6]This adjustment cost function is similar to those functions used by Hairault and Portier (1993), Ireland (1997), and Kim (2000).

[7]Burnside, Eichenbaum, and Rebelo (1993) assume that employment-adjustment costs are infinite in the current period, so they introduce labour hoarding into a model where the good market is perfectly competitive.

gradually and intertemporally smooth their labour demand (see Hamermesh 1993, chapter 5). The specification form in (20), which assumes that the marginal cost of adjusting employment is a linear function of its rate of change, was also used by Cogley and Nason (1995) when they studied the output dynamic propagation in a model with finite capital and employment-adjustment costs.

Price- and employment-adjustment costs make the representative intermediate-good-producing firm's problem dynamic. The problem of firm j is to choose contingency plans for h_{jt}, k_{jt}, y_{jt}, and p_{jt}, $t = 0, \cdots, \infty$, that maximize its expectation of the discounted sum of its profit flows conditional on the information available at time zero:

$$\max_{\{k_{jt}, h_{jt}, p_{jt}\}} E_0 \left[\sum_{t=0}^{\infty} \beta^t \lambda_t D_{jt} / p_t \right], \tag{21}$$

where the instantaneous profit function is given by

$$D_{jt} = p_{jt} y_{jt} - p_t r_t k_{jt} - p_t w_t h_{jt} - p_t PAC_{jt} - p_t EAC_{jt}, \tag{22}$$

subject to constraints (15) and (17). The firm's discount factor is given by the stochastic process $(\beta^t \lambda_t)$, where λ_t denotes the marginal utility of real wealth. In equilibrium, this factor represents a pricing kernel for contingent claims. The first-order conditions are derived from the following Bellman equation:

$$V\left(h_{jt-1}, p_{jt-1}, \Omega_t\right) = \max_{\{k_{jt}, h_{jt}, p_{jt}\}} \left\{ \lambda_t D_{jt} / p_t + \beta E_t \left[V\left(h_{jt}, p_{jt}, \Omega_{t+1}\right) \right] \right\}$$

subject to

$$A_t k_{jt}^{\alpha} \left(g^t h_{jt}\right)^{1-\alpha} \geq \left(\frac{p_{jt}}{p_t} \right)^{-\theta} y_t, \tag{23}$$

to which the Lagrangian multiplier $\xi_t > 0$ is associated. Ω_t is the information set upon which expectations formed in period t are conditioned. The first-order conditions with

respect to k_{jt}, h_{jt}, p_{jt}, and ξ_t are, respectively, given by

$$\alpha \frac{y_{jt}}{k_{jt}} \frac{\xi_t}{\lambda_t} - r_t = 0; \tag{24}$$

$$(1-\alpha)\frac{y_{jt}}{h_{jt}}\frac{\xi_t}{\lambda_t} - w_t - \phi_h \left(\frac{h_{jt}}{h_{jt-1}} - 1\right)\frac{y_t}{h_{jt-1}} + \beta\phi_h E_t\left[\left(\frac{h_{jt}}{h_{jt-1}} - 1\right)\frac{h_{jt+1}y_{t+1}}{h_{jt}^2}\frac{\lambda_{t+1}}{\lambda_t}\right] = 0; \tag{25}$$

$$\frac{\xi_t}{\lambda_t} - \frac{\theta-1}{\theta} - \frac{\phi_p}{\theta}\left(\frac{p_{jt}}{p_{jt-1}} - 1\right)\frac{p_{jt}}{p_{jt-1}}\frac{y_t}{y_{jt}} + \frac{\beta\phi_p}{\theta}E_t\left[\left(\frac{p_{jt+1}}{p_{jt}} - 1\right)\frac{p_{jt+1}}{p_{jt}}\frac{\lambda_{t+1}}{\lambda_t}\frac{y_{t+1}}{y_{jt}}\right] = 0; \tag{26}$$

$$A_t k_{jt}^{\alpha}\left(g^t h_{jt}\right)^{1-\alpha} - \left(\frac{p_{jt}}{p_t}\right)^{-\theta} y_t = 0. \tag{27}$$

Equations (24) and (25) equate the marginal rate of substitution in production between capital and labour to their relative price. With employment-adjustment costs, the price of labour consists of the real wages paid to the household and the marginal cost of adjusting labour in the current and future periods. Equation (26) governs the adjustment of the intermediate-good-producing firm's nominal price over time.

Equations (24) and (25) indicate that $q_t = \lambda_t/\xi_t$ measures the gross price-markup over marginal cost. In the absence of price-adjustment costs ($\phi_p = 0$), equation (26) implies that the markup is constant and equal to $\theta/(\theta-1)$. That is, the marginal cost does not adjust in response to exogenous disturbances.

With nominal price rigidities, the markup varies in response to exogenous disturbances. For example, following a positive technology shock, the marginal cost curve shifts downward, and since the intermediate-good-producing firm does not fully adjust its price, both the markup and output increase. On the other hand, a positive aggregate demand shock shifts the marginal revenue curve upward, and, given that prices are sticky, the markup decreases, while labour demand and output increase.

With nominal and real rigidities, labour demand gradually increases following a positive money supply shock, inducing a slower adjustment of real wages. Since the marginal

cost depends on real wages, it also changes more slowly.

2.4 The monetary authority

Following Ireland (1997) and Galí (1999), where the monetary authority systematically responds to random shocks affecting the economy, I assume that monetary policy can be endogenous. Hence, the monetary authority can adjust the nominal money supply in response to technology and money demand shocks. Therefore, the central bank manages the nominal money stock by making lump-sum transfers to the representative household during each period, so that

$$M_t - M_{t-1} = T_t, \tag{28}$$

where M_t is the per capita money stock. Monetary policy evolves according to the rule:

$$\log(\mu_t) = (1 - \rho_\mu)\log(\mu) + \rho_\mu \log(\mu_{t-1}) + \omega_A \varepsilon_{At} + \omega_b \varepsilon_{bt} + \varepsilon_{\mu t}, \tag{29}$$

where $\mu_t = \frac{M_t}{M_{t-1}}$ denotes the gross growth rate of money in period t, $\rho_\mu \in (-1, 1)$, and $\varepsilon_{\mu t}$ is a serially uncorrelated money supply shock that is normally distributed with zero mean and standard deviation σ_μ. The money supply shock is uncorrelated with the money demand and technology shocks at all leads and lags. In the event that $\omega_A = \omega_b = 0$, monetary policy becomes purely exogenous.

2.5 Symmetric equilibrium and resolution

In a symmetric equilibrium, all intermediate-good-producing firms are identical. They make the same decisions, so that $k_{jt} = k_t$, $h_{jt} = h_t$, $p_{jt} = p_t$, $y_{jt} = y_t$, and $D_{jt} = D_t$. Let $\pi_t = p_t/p_{t-1}$ denote the inflation rate in period t. The symmetric equilibrium is composed of an allocation, $\{y_t, c_t, M_t/p_t, h_t, k_t, i_t\}_{t=0}^{\infty}$, and a sequence of prices and co-state variables, $\{w_t, r_t, \pi_t, \lambda_t, q_t\}_{t=0}^{\infty}$, that satisfy the household's first-order conditions (8) to (12), the intermediate-good-producing firm's first-order conditions (24) to (27), the aggregate resource constraint, the money supply rule (28), and the stochastic processes

of money demand, technology, and money supply shocks, equations (3), (18), and (29).[8] This system is composed of 14 equations and 14 variables (a detailed description is given in Appendix A).

In the model, h_t, r_t, π_t, q_t, A_t, b_t, and μ_t are stationary variables. The remaining variables must be made stationary by defining the following:

$$\tilde{c}_t = \frac{c_t}{g^t}, \tilde{m}_t = \frac{M_t/p_t}{g^t}, \tilde{k}_t = \frac{k_t}{g^t}, \tilde{w}_t = \frac{w_t}{g^t}, \tilde{y}_t = \frac{y_t}{g^t}, \tilde{i}_t = \frac{i_t}{g^t}, \text{ and } \tilde{\lambda}_t = \frac{\lambda_t}{g^{-t}}.$$

Taking these definitions into account and given $\tilde{k}_0, \tilde{m}_{-1}, h_{-1}$, and $\{A_t, b_t, \mu_t\}_{t=0}^{\infty}$, one obtains equilibrium conditions for the allocation $\left\{\tilde{y}_t, \tilde{c}_t, \tilde{m}_t, h_t, \tilde{k}_t, \tilde{i}_t\right\}_{t=0}^{\infty}$ and the sequence of prices and co-state variables $\left\{\tilde{w}_t, r_t, \pi_t, \tilde{\lambda}_t, q_t\right\}_{t=0}^{\infty}$ (the transformed system describing the stationary equilibrium is given in detail in Appendix B).

The log-linear approximation of the equilibrium system around steady-state values is obtained by using the methods described in Blanchard and Kahn (1980) and King, Plosser, and Rebelo (1987). For any stationary variable $\tilde{x}_t$, I define $\hat{x}_t = \log(\tilde{x}_t/\tilde{x})$ as the deviation of $\tilde{x}_t$ from its steady-state value (see Appendix C). The log-linearized version of the model can thus be written in its state-space form:

$$\hat{s}_{t+1} = \Phi_1 \hat{s}_t + \Phi_2 \varepsilon_{t+1}, \tag{30}$$

$$\hat{d}_t = \Phi_3 \hat{s}_t, \tag{31}$$

where $\hat{s}_t = \left(\hat{k}_t, \hat{m}_{t-1}, \hat{h}_{t-1}, \hat{A}_t, \hat{b}_t, \hat{\mu}_t\right)'$ is a vector of state variables that includes predetermined and exogenous variables; $\hat{d}_t = \left(\hat{\lambda}_t, \hat{q}_t, \hat{m}_t, \hat{h}_t, \hat{y}_t, \hat{w}_t, \hat{r}_t, \hat{c}_t, \hat{\pi}_t, \hat{i}_t\right)'$ is the vector of control variables; and the vector $\varepsilon_{t+1} = (\varepsilon_{At+1}, \varepsilon_{bt+1}, \varepsilon_{\mu t+1})'$ contains technology, money demand, and money supply shocks. The solution is a restricted vector autoregression (VAR) in the sense that the coefficient matrices, Φ_1, Φ_2, and Φ_3, depend on

[8] The aggregate resource constraint is derived directly from the household's budget constraint. Market clearing for money requires $M_t - M_{t-1} = T_t$. Substituting this condition and the equation defining the intermediate-good-producing firm's profit, D_t, into the household's budget equation yields the aggregate resource constraint.

the structural parameters of the model.[9] Using the system (30) - (31), I estimate the underlying structural parameters and simulate the model.

3 Calibration and Estimation Procedures

3.1 Calibration procedure

There are 22 structural parameters in the model. Seven are fixed prior to estimation, as the data used contain only limited information about them. The discount factor β is set at 0.992, implying a steady-state real interest rate of 3 per cent. The parameter η, denoting the weight put on leisure in the representative household's utility function, is set at 1.42, so that the representative household spends roughly one third of its time in market activities. The parameter b, determining the steady-state ratio of real balances to consumption, is set equal to 0.535, matching the steady-state consumption velocity of money in the model to the average consumption velocity of M2 in the Canadian data from 1976 to 2000. I set ϕ_k, the capital adjustment cost parameter, equal to 1, which produces an average cost of capital adjustment of about 0.3 per cent of quarterly GDP. This value is consistent with the accepted notion that capital adjustment costs are economically significant but small (see Mendoza 1991).[10] The depreciation rate, δ, and the share of capital in production, α, are assigned values of 0.025 and 0.33, respectively.[11] Finally, θ, which measures the degree of monopoly power in the intermediate-good markets, is set equal to 6, so that the gross steady-state markup of price over marginal cost in the model matches the benchmark value of 1.2 in Rotemberg and Woodford (1995). Table 1 summarizes the values that have been assigned to these parameters.

[9]The solution is equivalent to the following relation in the non-transformed variables:

$$\log(X_t) = [(I - \Phi_1)\log(x) + \Phi_1 \log(g)] + [(I - \Phi_1)log(g)]t + \Phi_1 log(X_{t-1}) + \Phi_2 \varepsilon_t,$$

which is a first-order VAR with a constant and a linear time trend.

[10]Based on an estimated DSGE model, Kim (2000) finds that the capital adjustment costs represent 5 per cent of investment in the U.S. economy.

[11]Mendoza (1991) uses these values to calibrate the depreciation rate and the share of capital for the Canadian economy.

Table 1: Calibrated parameter values

Parameters	β	η	b	ϕ_k	δ	α	θ
Values	0.992	1.42	0.535	1.0	0.025	0.33	6.0

The estimates of the price- and employment-adjustment cost parameters are slightly affected by the choice of ϕ_k and θ. The degree of the nominal rigidity increases with ϕ_k; however, the estimated values for ϕ_h decrease when ϕ_k increases. Price adjustment becomes more rapid when θ increases (that is, when the markets become more competitive), so price-adjustment costs become larger.[12]

3.2 Estimation procedure

The remaining structural parameters are estimated using the method of Hansen and Sargent (1998), which consists of applying the Kalman filter to the state-space form of the model to generate a series of innovations, $\{\varepsilon_t\}_{t=1}^{T}$, that are used to evaluate the likelihood function for the sample. Since the solution in (30)-(31) is a restricted first-order VAR, in the sense that the coefficient matrices, Φ_1,Φ_2, and Φ_3, are non-linear functions of deep parameters, the parameters of the model can be estimated by maximizing this likelihood function (see also Hamilton 1994, chapter 13).

Using a first-order VAR for the detrended output, the inflation rate, and the money growth rate, I estimate this DSGE model using quarterly Canadian data from 1976Q1 to 2000Q4. The output is real per capita GDP, the price level is the implicit GDP deflator, and the nominal money stock is measured by M2 per capita. To obtain a per capita variable, I divide the variable by the total civilian non-institutional population aged 15 and over.

In the Canadian data, the inflation and the nominal M2 growth rate are not stationary; they grow at a rate g_π, which is less than 1.[13] I transform these two variables into

[12] I also estimate the model with $\phi_k = 3$ and $\theta = 9$, and the results confirm the above intuition.

[13] Thus, they exhibit a negative linear trend.

stationary variables using the relations $\pi_t = \pi_t^d / g_\pi^t$ and $\mu_t = \mu_t^d / g_\pi^t$, where π_t^d and μ_t^d are inflation and money growth series. The parameter g_π is also estimated with the structural parameters of the model.

Let $\vartheta = (\phi_p, \phi_h, \gamma, b, \rho_b, \sigma_b, g, A, \rho_A, \sigma_A, \mu, \rho_\mu, \omega_A, \omega_b, \sigma_\mu, g_\pi)'$ be a 16-vector of structural parameters to estimate. Assuming normality of the innovations ε_t and making use of the Kalman filter, I maximize the following maximum log-likelihood function:

$$l(\varepsilon, \vartheta) = \frac{T}{2} \ln |\Sigma| + \frac{1}{2} \sum_{t=1}^{T} \varepsilon_t' \Sigma^{-1} \varepsilon_t, \tag{32}$$

where $\varepsilon_t \sim N(0, \Sigma)$ and Σ is the variance-covariance matrix that depends on the structural parameters of the model.[14]

Equation (32) is maximized subject to the elements of the vector ϑ that are included in the structural parameter matrices, Φ_1, Φ_2, and Φ_3. As Hansen and Sargent (1998) show, the ML estimator is consistent and asymptotically efficient. The Kalman filter updates the estimator as new information becomes available, thus ensuring an optimal prediction.

3.3 Estimation results

In this section, I describe the estimation results of the structural parameters for a standard sticky-price model (SSP model), where employment and capital are flexible, and for the other three versions of the DSGE model with nominal and real rigidities. The first version is a model with price and capital rigidities (PCR model). The second is a model with price and employment rigidities (PER model), where it is costly to adjust labour, while the capital is perfectly flexible. The third version is a model with price, capital, and employment rigidities (PCER model).[15] Table 2 reports maximum-likelihood estimates and their standard errors for these four models.

Almost all parameter estimates are highly significant at conventional confidence levels, consistent, and economically meaningful. The estimate of the price-adjustment cost

[14] I ignore the constant term.

[15] In the SSP model, $\phi_k = \phi_h = 0$; in the PCR model, $\phi_h = 0$; and in the PER model, $\phi_k = 0$.

parameter, ϕ_p, is significantly larger when both nominal and real frictions are considered; the estimate of ϕ_p is 2.80 in the SSP model, while it is 14.36 (44.07) in the PCR model (and the PER model), and 26.63 in the PCER model. With $\phi_p = 26.63$, changing nominal prices by 1 per cent involves paying a cost that amounts to about 0.13 per cent of real GDP per quarter. This cost is quite similar to that postulated by Hairault and Portier (1993) for the U.S. and French economies, but it is larger than the one estimated by Kim (2000) and Ireland (1997) for the U.S. economy.[16] Nonetheless, using post-war U.S. data, Dib and Phaneuf (2001) estimate ϕ_p equal to 4.26 in the SSP model and about 93.0 in their model with price- and employment-adjustment costs.

The employment-adjustment cost parameter, ϕ_h, is estimated at 1.85 in the PER model and at 0.44 in the PCER model, but with relatively high standard errors. Hence, the marginal cost of changing employment by 1 per cent amounts to roughly 1.85 per cent of real GDP per quarter when the capital is flexible, but only about 0.44 per cent of a quarter's output in the sticky-price model with the capital and employment rigidities. The degree of employment rigidity decreases with the presence of capital adjustment costs. Therefore, the estimated value for ϕ_h is smaller in the model that combines both types of real rigidities. Using Shapiro's (1986) estimates for the U.S. economy to calibrate the employment-adjustment cost parameter, Cogley and Nason (1995) set ϕ_h equal to 0.36 in their model with capital and employment rigidities.

The estimate of γ, the constant elasticity of substitution between real consumption and real balances, implies an interest elasticity of money demand equal to -0.30 in the SSP model, while it is estimated at about -0.40 when both nominal and real rigidities are combined. Money demand shocks are highly persistent, and quite large and volatile. The estimated values of the autoregressive coefficient, ρ_b, exceed 0.99 in all models, while the unconditional standard error, σ_b, is estimated at 0.022. These estimates are similar to those estimated by Ireland (1997), Kim (2000), and Dib and Phaneuf (2001) for the U.S.

[16] In their calibration, Hairault and Portier (1993) assume that price-adjustment costs represent 0.1 per cent of quarterly output. Kim (2000) estimates that the price-adjustment cost parameter is 0.806, but in the presence of adjustment costs of nominal wages. However, Ireland's (1997) estimate is 4.05 in his standard sticky-price model.

economy.

The estimated values of $g = 1.0033$ and $A = 220$ imply that the annual trend rate of real per capita GDP growth is about 1.40 per cent, and that the level of per capita real output in the model and in the data is quite similar. The technology shocks appear to be highly persistent, with ρ_A estimated at about 0.95 in the four models. The unconditional standard deviation, σ_A, is estimated at 0.0057 in the SSP model, and at 0.0064 in the models with nominal and real rigidities. The values estimated for the technology processes parameters are similar to those normally assumed in real business cycle (RBC) studies.

The estimate of the money growth rate, μ, is 1.024 in all models. However, the serial correlation in the money growth process, ρ_μ, is 0.74 in the SSP model, and it ranges between 0.70 in the PCR model and 0.76 in the PER model. The variation in money growth not explained by the central bank's endogenous response to technology and money demand shocks, σ_μ, is estimated at 0.0015, which is small compared to that estimated directly by an unconstrained VAR.[17] The estimates of ω_A, the parameter that measures the response of the monetary authority to technology shocks, is estimated at 0.15 in the two models where labour is perfectly flexible; however, it is estimated at 0.06 and 0.11 in the PER and PCER models, respectively. On the other hand, the parameter measuring the response of monetary policy to money demand shocks, ω_b, is estimated at about 0.30 in all models. The response to technology shocks has been procyclical, while it has been counter-cyclical with respect to money demand shocks. The larger value estimated for ω_b indicates that the monetary authority aggressively responds to money demand shocks to smooth the inflation rate. Finally, the steady-state quarterly inflation rate is determined as $\pi = \mu/g = 1.0209$.

The estimate of the growth rate of inflation and nominal money growth, g_π, is 0.9998 in all estimated models. This value indicates that inflation and nominal money growth

[17]Under the hypothesis of exogenous monetary policy, the estimate of σ_μ is about 0.0063. However, under the hypothesis of endogenous monetary policy, the variance of the money growth rate is equal to $var(\mu_t) = \frac{\omega_A^2\sigma_A^2+\omega_b^2\sigma_b^2+\sigma_\mu^2}{1-\rho_\mu^2}$. This explains why the estimate of σ_μ is smaller in the endogenous monetary policy models.

have decreased by about 0.0002 per quarter during the sample period.[18]

4 Evaluating the Model

This section evaluates the performance of the four alternative models using the estimated and calibrated values for their structural parameters. Furthermore, to study the implications of the partially endogenous monetary policy hypothesis, I evaluate an alternative exogenous economy, where the parameters ω_A and ω_b are set equal to 0, and σ_μ is equal to 0.0063, keeping the other structural parameters at their estimated and calibrated values in the PCER model.[19] Thus, the central bank no longer responds to technology and money demand shocks.

Based on the estimated models and the exogenous economy and using the state-space form in (30)-(31), I first calculate the impulse-response functions of output, real wages, hours, and the inflation rate to money supply shocks, technology shocks, and money demand shocks. The impulse responses are computed for 1 per cent shocks and expressed as the percentage deviation of a variable from its steady-state value. I then calculate the forecast-error variance decomposition of detrended output, inflation, and the money growth at various horizons. Next, I calculate the standard deviations generated by the data and the various models.

4.1 Impulse-response functions

4.1.1 Responses to money supply shocks

Figure 1 shows the impulse responses for a 1 per cent increase in the money growth rate in the four estimated models. In the SSP model, where there is only the nominal rigidity, output, real wages, hours, and the inflation rate immediately jump above their steady-state values, but their responses exhibit no significant persistence. However, the combination of nominal and real rigidities imparts a substantial degree of nominal price

[18] As g_π is less than 1, $\ln(g_\pi) = -0.0002$.

[19] Under the hypothesis of exogenous monetary policy, the estimate of σ_μ is equal to 0.0063 in the VAR.

rigidity. Indeed, in the models that have both nominal and real rigidities, output, real wages, and hours increase by more than 2.8 per cent during the period of the shock, and their responses last for more than seven quarters after the shock. Even though the immediate effect of a money supply shock on the inflation rate is positive and highly significant, its response exhibits little persistence; it dies out at the end of the fourth quarter.

The real monetary shock effects are more significant and persistent in the presence of employment rigidity. This merely reflects the slow adjustment of real wages and employment to their steady-state level in response to a money supply shock. Indeed, in the presence of employment-adjustment costs, intermediate-good-producing firms have to pay a cost for changing the quantity of the labour input. They also have to pay much larger price-adjustment costs. As a consequence, they remain reluctant to change their prices in response to money supply shocks. Nonetheless, the employment-adjustment costs play a significant role because, once the intermediate-good producers have allowed output to increase in response to a positive shock to demand, firms are less willing to lower output immediately. Thus, output movements are more persistent. Overall, these results confirm that nominal rigidities combined with real frictions are an important source of the real persistence.

Figure 2 shows how the economy responds to a 1 per cent money supply shock in the SSP model. To test whether more persistence can be obtained if the size of price-adjustment costs is increased, I set ϕ_p equal to 26.63, as estimated in the PCER model, and keep the other structural parameters at their estimated and calibrated values for the SSP model. The nominal rigidity produces a large output effect, but no persistence. Although the contemporaneous responses of the variables are stronger with larger price-adjustment costs, the real money effects do not exhibit more persistence. Hairault and Portier (1993), Ireland (1997), Kim (2000), and Dib and Phaneuf (2001) find similar results for the U.S. and French economies.

The results presented in this subsection confirm the idea stressed by CKM (2000), in which a model with sticky prices is unable to reproduce enough real effects from

nominal disturbances. However, as stressed by Dib and Phaneuf (2001), if one follows Ball and Romer's (1990) original insight of combining nominal and real frictions, the extent of nominal price rigidity can be substantially magnified so that aggregate prices may adjust slowly in response to money supply shocks, inducing output persistence.

4.1.2 Responses to technology shocks

Figure 3 shows the impulse responses to a 1 per cent positive technology shock computed in the four estimated models. In the SSP and PCR models, output, real wages, and hours immediately jump in response to the technology shock before returning gradually to their steady-state levels; however, these real variables increase gradually in the presence of employment-adjustment costs (in the PER and PCER models). The response of inflation, which is counter-cyclical in response to technology shocks, is negative and exhibits little persistence.

Figure 4 shows the impulse responses to a 1 per cent positive technology shock in the estimated PCER and exogenous PCER models. In the estimated PCER model, where the monetary authority accommodates supply shocks, output, real wages, and hours gradually increase after a positive technology shock. Inflation, on the other hand, responds negatively, but less significantly, before returning to its steady-state level.

In the exogenous PCER model, where the monetary authority does not respond to supply shocks, the immediate response of hours worked and inflation is negative and significant. Thus, technology shocks imply more volatility in the exogenous economy.

Since in the estimated PCER model the monetary authority accommodates supply shocks, a significant fraction of the downward price pressure from the supply shock is offset by upward pressure from an increase in the money supply. Therefore, the decline in hours worked is not significant. However, in the exogenous PCER model, the decrease in nominal price is considerable, leading to a significant decline in hours worked.

Given that the amount of nominal price rigidity imparted by the nominal and real frictions is very substantial, the intermediate-good-producing firms can meet their demand with less labour input given the increase in labour productivity. Notice also that,

as the evidence reported in Galí (1999) suggests, the decline in hours lasts approximately three quarters.

4.1.3 Responses to money demand shocks

Figure 5 shows the impulse responses to a 1 per cent positive money demand shock in the four estimated models. Because the monetary authority responds to unfavourable money demand shocks by increasing the money supply, the responses of output, real wages, and hours are negative but systematically very small. Hence, it appears that the monetary authority successfully insulates the economy from the effects of exogenous money demand shocks by responding counter-cyclically to these shocks. Inflation positively responds to money demand shocks, and its response peaks two quarters after the shock and persists for more than two years. However, the deviation of inflation is quite small as well.

Figure 6 shows the impulse responses to a 1 per cent positive money demand shock in the estimated and exogenous PCER models. In the exogenous model, where the monetary authority fails to respond to the shock, output, real wages, and hours decrease substantially during the shock period, and their negative responses last for about five quarters. The inflation rate immediately and significantly jumps down before returning to its steady-state value in the next quarter.

Since all real variable responses to money demand shocks are systematically very small in the estimated models, one can conclude that the monetary authority successfully insulates the economy from the effects of exogenous money demand shocks by responding counter-cyclically to these shocks. Ireland (1997) and Dib and Phaneuf (2001) find similar results for the U.S. economy.

4.2 Variance decomposition

Another way to look at the implications of the nominal and real rigidities is by computing the fractions of the forecast-error variance of detrended output, the inflation rate, and the money growth rate attributable to each type of shock. The results of this decomposition

for several forecast horizons are reported in Tables 3 to 7 for the four estimated models and the exogenous PCER model.

Table 3 gives the forecast-error variance decomposition of detrended output, inflation, and money growth in the SSP model, where there is only price rigidity. As shown in Panel A, money supply and money demand shocks explain a very small fraction of output fluctuations even in the short term. However, technology shocks largely contribute to the variations of detrended output in the short and long term. This result matches the standard RBC models' predictions, in which technology shocks are the most important factor for output fluctuations in the short and long term. Panel B and Panel C show that the SSP model predicts that money supply shocks are the most important source of inflation fluctuations in the short and long term, and money demand shocks explain about 94 per cent of the forecast-error variance of the money growth.

Tables 4 to 6 report the forecast-error variance decomposition of detrended output, inflation, and money growth in the estimated models with nominal and real rigidities (PCR, PER, and PCER models, respectively). As shown in Panel A of the tables, money supply shocks contribute very substantially to the variance of detrended output at short horizons. At the one-quarter-ahead horizon, money supply shocks account for at least 37 per cent of the forecast-error variance of detrended output. Furthermore, up to the one-year-ahead horizon, money supply shocks account for at least 17 per cent of the forecast-error variance. Even at the ten-quarter-ahead horizon, money supply shocks still explain close to 7 per cent of the variance of output. The counterpart, of course, is that technology shocks contribute less to short-run variation. At the one- and two-quarter-ahead horizons, technology shocks explain more than 47 per cent and 65 per cent of forecast-error variance of detrended output. By the tenth quarter, technology shocks account for at least 90 per cent. This result is in contrast with the standard prediction of RBC models, which assumes that real disturbances explain almost all output fluctuations in the short and long terms. The contribution of money demand shocks is negligible, thus corroborating our impulse-response analysis.

Panel B in Tables 4 to 6 show the forecast-error variance decomposition of the in-

flation rate. In this case, the models combining nominal and real rigidities predict that money supply shocks are the important factor determining movements in the inflation rate at short and longer horizons. The money supply shock accounts for nearly 93 per cent of the variance of the inflation rate at the one-quarter-ahead horizon, and it still contributes at least 57 per cent at the ten-quarter-ahead horizon. Technology shocks negligibly contribute to the variance of inflation in the short term, with only about 4 to 6 per cent at the one- and four-quarter-ahead horizons, but they substantially contribute to the inflation variance in the long term, with about 27 per cent and 87 per cent at the ten- and fifty-quarter-ahead horizons, respectively. The contribution of money demand shocks is negligible.

Panel C in Tables 4 to 6 show that more than 92 per cent of the money growth variance is explained by money demand shocks. Similar results are found by Rotemberg and Woodford (1997), Ireland (1997), and Dib and Phaneuf (2001) for the U.S. economy.

Table 7 presents the forecast-error variance decomposition of detrended output, inflation, and money growth in the exogenous PCER model. Panel A shows that money supply and money demand shocks explain a major part of the short-term forecast-error variance of detrended output. They account for as much as 43 per cent and 56 per cent one-quarter ahead, and 35 per cent and 46 per cent ten-quarters-ahead, respectively. In contrast, technology shocks explain only a small fraction in the short term. In Panel B, money supply and money demand shocks also explain, together, more than 98 per cent of forecast-error variance of inflation in the short and long term, while technology shocks account for only a small fraction. In Panel C, all of the money growth fluctuations are explained by money supply shocks, owing to the fact that monetary policy is perfectly exogenous.

4.3 Output, inflation, and money growth volatility and autocorrelation

Ellison and Scott (2000) show that the sticky-price models not only fail to produce persistent business-cycle fluctuations, but they generate extreme volatility in output and

inflation. Using the estimated and exogenous PCER models, I calculate the standard deviations and autocorrelation coefficients of the detrended output, inflation, and money growth. Table 8 reports the standard deviations, expressed in percentage terms, and the autocorrelations as computed in the data and in the various models.

The estimated models slightly overpredict the volatility of output; however, the inflation and money growth are as volatile in the models as in the data. Nonetheless, the exogenous PCER model generates a high standard deviation of output and extreme volatility in inflation. Overall, the estimated models are able to reproduce acceptable volatility for the main variables of the DSGE model. These results indicate that the monetary authority succeeds in reducing the output and inflation volatility by managing an endogenous monetary policy.

The data also report that detrended output, inflation, and money growth are positively and very significantly autocorrelated over short horizons. At a lag of one quarter, the four estimated models (SSP, PCR, PER, and PCER) are able to generate the observed autocorrelation of detrended output; however, at lags of 2 and 3 quarters, the generated autocorrelations are much greater than those of the sample. The detrended output is highly persistent in the estimated models, but it is less persistent in the exogenous PCER model. On the other hand, the models fail to account for the high inflation persistence observed in the data; the generated first-order autocorrelation is less than 0.29, compared to 0.93 in the data. Thus, there is very little sluggishness in the inflation rate. The money growth autocorrelations obtained in the estimated models resemble those computed in the data. The obtained first-order autocorrelation is at least 0.70. The higher value reflects the fact that I use M2 as the definition of money and that the monetary authority reacts to contemporaneous technology and money demand shocks.

5 Conclusion

Ever since the monumental study by Friedman and Schwartz (1963), it has been repeatedly confirmed that nominal disturbances exert a significant impact on economic

fluctuations, at least in the short term. One piece of evidence that has been difficult to explain within the recently developed models emphasizing the optimizing behaviour of rational agents is the persistent real effect of monetary policy shocks. I have shown in this paper that combining nominal and real rigidities can substantially magnify the dynamic propagation of money supply shocks in a DSGE environment.

The structural parameters that are essential for the identification of transmission channels leading to high real persistence have been the object of econometric estimation using Canadian quarterly data from 1976Q1 to 2000Q4. According to the results, the combination of nominal and real rigidities induces higher estimated values of the price-adjustment cost parameter which, in turn, implies a greater amount of nominal price rigidity, and generates higher output persistence in response to money supply shocks. Given that the estimated degree of monetary accommodation is high, the higher degree of nominal price rigidity implies that hours worked slightly decline in response to a positive technology shock. Furthermore, adding real rigidities to the standard sticky-price model with partially endogenous monetary policy significantly reduces output and inflation volatility.

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Table 2:
Maximum-likelihood estimates and standard errors: 1976Q1 to 2000Q4

Parameters	SSP model	PCR model	PER model	PCER model
ϕ_p	2.8048	14.356	44.073	26.626
	(2.5097)	(4.4216)	(25.288)	(10.213)
ϕ_h	-	-	1.8538	0.4377
	(-)	(-)	(1.0377)	(0.2955)
γ	0.3005	0.3887	0.3940	0.4366
	(0.1174)	(0.1687)	(0.2537)	(0.2504)
ρ_b	0.9998	0.9999	0.9997	0.9998
	(0.0055)	(0.0053)	(0.0060)	(0.0056)
σ_b	0.0210	0.0200	0.0241	0.0222
	(0.0043)	(0.0034)	(0.0071)	(0.0047)
g	1.0033	1.0034	1.0033	1.0034
	(0.0003)	(0.0004)	(0.0003)	(0.0003)
A	218.04	219.98	217.98	220.67
	(2.8323)	(3.9294)	(5.5045)	(6.4944)
ρ_A	0.9578	0.9592	0.9310	0.9471
	(0.0259)	(0.0298)	(0.0280)	(0.0276)
σ_A	0.0057	0.0064	0.0064	0.0064
	(0.0005)	(0.0006)	(0.0005)	(0.0005)
μ	1.0235	1.0243	1.0238	1.0240
	(0.0015)	(0.0013)	(0.0015)	(0.0015)
ρ_μ	0.7370	0.7039	0.7592	0.7321
	(0.0534)	(0.0485)	(0.0572)	(0.0498)
σ_μ	0.0015	0.0017	0.0013	0.0015
	(0.0003)	(0.0003)	(0.0003)	(0.0003)
ω_A	0.1480	0.1498	0.0616	0.1072
	(0.0438)	(0.0477)	(0.0419)	(0.0431)
ω_b	0.3225	0.3347	0.2854	0.3054
	(0.0610)	(0.0528)	(0.0793)	(0.0608)
g_π	0.9998	0.9998	0.9998	0.9998
	(0.00003)	(0.00003)	(0.00003)	(0.00003)

Table 3:
Forecast-error variance decomposition in the SSP model

Quarters	Variance	Percentage owing to:		
		Money supply	Technology	Money demand
		A. Detrended output		
1	0.000067	5.47	92.06	2.47
2	0.000128	2.86	95.67	1.46
3	0.000187	1.97	96.90	1.12
4	0.000243	1.51	97.53	0.86
5	0.000297	1.24	97.90	0.86
10	0.000532	0.70	98.59	0.71
50	0.001176	0.32	98.18	150
		B. Inflation		
1	0.0000196	99.32	0.66	0.02
2	0.0000216	90.78	0.64	8.58
3	0.0000228	86.57	0.84	12.59
4	0.0000237	83.37	2.03	14.61
5	0.0000248	79.96	4.62	15.42
10	0.0000368	54.27	34.00	11.73
50	0.000252	7.97	89.96	2.07
		C. Money growth		
1	0.00005	4.61	1.46	93.93
2	0.00007	4.61	1.46	93.93
5	0.00010	4.61	1.46	93.93
50	0.00011	4.61	1.46	93.93

Table 4:
Forecast-error variance decomposition in the PCR model

Quarters	Variance	Percentage owing to: Money supply	Technology	Money demand
		A. Detrended output		
1	0.00011	48.28	46.75	4.96
2	0.00018	30.54	66.31	3.15
3	0.00025	21.95	75.77	2.28
4	0.00033	17.02	81.16	1.81
5	0.00040	13.86	84.62	1.52
10	0.00080	7.12	91.87	1.00
50	0.00269	2.20	96.08	1.73
		B. Inflation		
1	0.000015	95.78	4.09	0.12
2	0.000016	92.03	3.96	4.02
3	0.000017	89.91	3.86	6.22
4	0.000018	88.34	4.35	7.30
5	0.000018	86.52	5.72	7.76
10	0.000024	66.90	26.72	6.38
50	0.000153	11.04	87.42	1.54
		C. Money growth		
1	0.00005	6.00	1.91	92.09
2	0.00007	6.00	1.91	92.09
5	0.00009	6.00	1.91	92.09
50	0.00010	6.00	1.91	92.09

Table 5:
Forecast-error variance decomposition in the PER model

Quarters	Variance	Percentage owing to: Money supply	Technology	Money demand
		A. Detrended output		
1	0.00006	37.39	60.90	1.71
2	0.00012	27.47	71.54	0.98
3	0.00018	21.06	78.27	0.66
4	0.00024	16.90	82.59	0.50
5	0.00030	14.09	85.50	0.40
10	0.00056	8.04	91.67	0.27
50	0.00119	4.02	92.90	3.08
		B. Inflation		
1	0.000020	93.32	6.46	0.22
2	0.000022	87.41	6.16	6.43
3	0.000023	84.22	5.96	9.81
4	0.000024	81.95	6.36	11.68
5	0.000025	79.50	7.86	12.63
10	0.000037	57.41	31.82	10.76
50	0.000182	12.71	84.30	2.98
		C. Money growth		
1	0.000049	3.97	0.31	95.72
2	0.000078	3.97	0.31	95.72
5	0.000104	3.97	0.31	95.72
50	0.000116	3.97	0.31	95.72

Table 6:
Forecast-error variance decomposition in the estimated PCER model

Quarters	Variance	Percentage owing to: Money supply	Technology	Money demand
		A. Detrended output		
1	0.00008	49.88	47.21	2.90
2	0.00015	33.40	64.74	1.86
3	0.00021	24.21	74.44	1.34
4	0.00028	18.77	80.16	1.06
5	0.00035	15.28	83.81	0.90
10	0.00068	8.05	91.16	0.76
50	0.00188	3.05	93.75	3.19
		B. Inflation		
1	0.000016	93.34	6.22	0.43
2	0.000017	88.49	6.02	5.49
3	0.000018	86.24	5.81	7.94
4	0.000019	84.60	6.17	9.23
5	0.000020	82.65	7.52	9.81
10	0.000027	62.39	29.55	8.05
50	0.000152	11.68	85.91	2.40
		C. Money growth		
1	0.000048	4.62	0.97	94.41
2	0.000075	4.62	0.97	94.41
5	0.000104	4.62	0.97	94.41
50	0.000105	4.62	0.97	94.41

Table 7:
Forecast-error variance decomposition in the exogenous PCER model

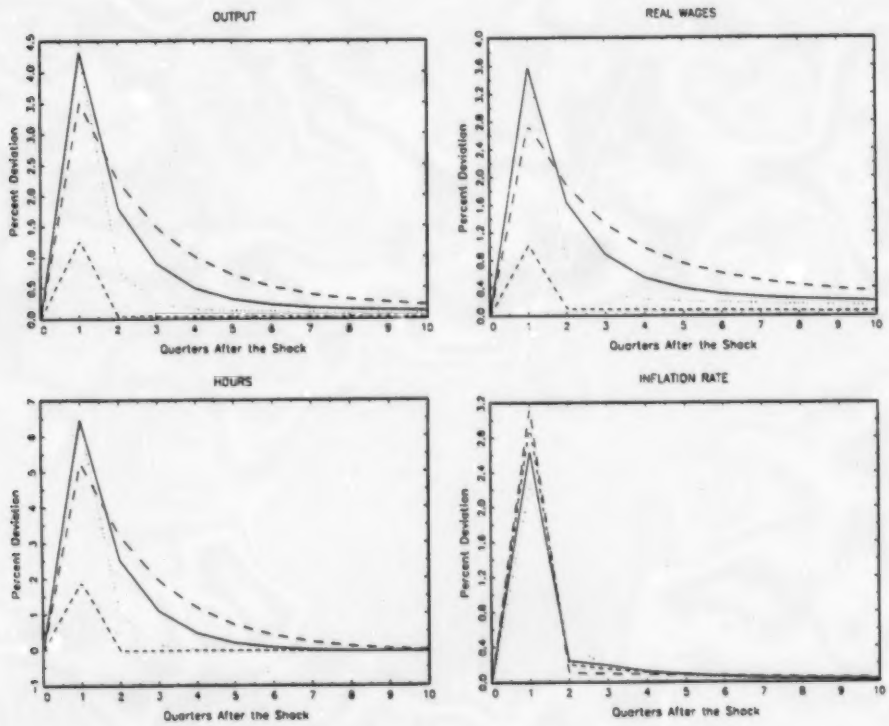
Quarters	Variance	Percentage owing to: Money supply	Technology	Money demand
		A. Detrended output		
1	0.00121	43.03	0.78	56.19
2	0.00146	42.14	2.91	54.94
3	0.00157	41.02	5.44	53.52
4	0.00164	39.86	8.05	52.09
5	0.00170	38.75	10.51	50.73
10	0.00192	34.64	19.63	45.73
50	0.00240	27.83	33.20	38.96
		B. Inflation		
1	0.000500	46.33	1.79	51.87
2	0.000508	46.66	1.80	51.53
3	0.000513	46.84	1.80	51.35
4	0.000514	46.92	1.80	51.27
5	0.000515	46.95	1.80	51.24
10	0.000515	46.99	1.80	51.20
50	0.000515	46.98	1.83	51.18
		C. Money growth		
1	0.000039	100	0.00	0.00
2	0.000061	100	0.00	0.00
5	0.000081	100	0.00	0.00
50	0.000085	100	0.00	0.00

Table 8:
Standard deviations and autocorrelations of detrended output, inflation, and money growth

	Data	SSP	SPC	SPE	SPEC	Exogenous
			A.Detrended output			
$sd(\tilde{y}_t)$	2.92	3.61	4.13	3.56	3.98	5.41
$cor(\tilde{y}_t, \tilde{y}_{t-1})$	0.96	0.97	0.95	0.97	0.97	0.71
$cor(\tilde{y}_t, \tilde{y}_{t-2})$	0.88	0.94	0.93	0.94	0.94	0.60
$cor(\tilde{y}_t, \tilde{y}_{t-3})$	0.78	0.92	0.90	0.91	0.91	0.54
			B. Inflation			
$sd(\pi_t)$	0.49	0.50	0.43	0.50	0.44	2.27
$cor(\pi_t, \pi_{t-1})$	0.93	0.21	0.29	0.18	0.21	0.14
$cor(\pi_t, \pi_{t-2})$	0.83	0.17	0.15	0.15	0.17	0.10
$cor(\pi_t, \pi_{t-3})$	0.70	0.13	0.11	0.13	0.13	0.05
			C. Money growth			
$sd(\mu_t)$	0.94	1.03	0.98	1.08	1.02	0.92
$cor(\mu_t, \mu_{t-1})$	0.67	0.74	0.70	0.76	0.73	0.73
$cor(\mu_t, \mu_{t-2})$	0.49	0.54	0.49	0.57	0.53	0.53
$cor(\mu_t, \mu_{t-3})$	0.45	0.40	0.35	0.44	0.39	0.44

Figure 1:

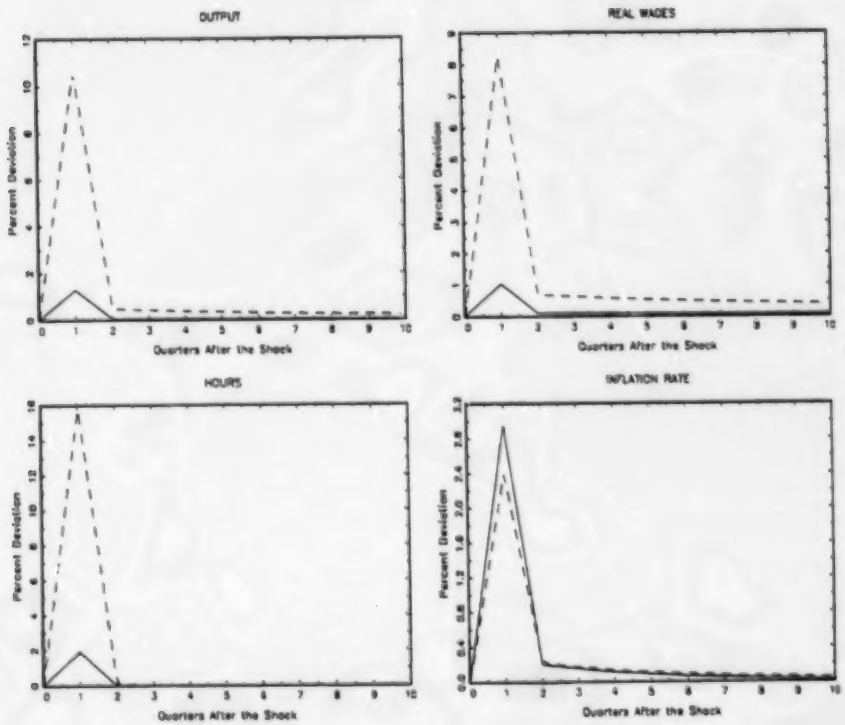
The effects of money supply shocks in the four estimated models



The impulse responses are computed for the SSP model (dashed line), the PCR model (dotted line), the PER model (large dashed line), and the PCER model (solid line).

Figure 2:

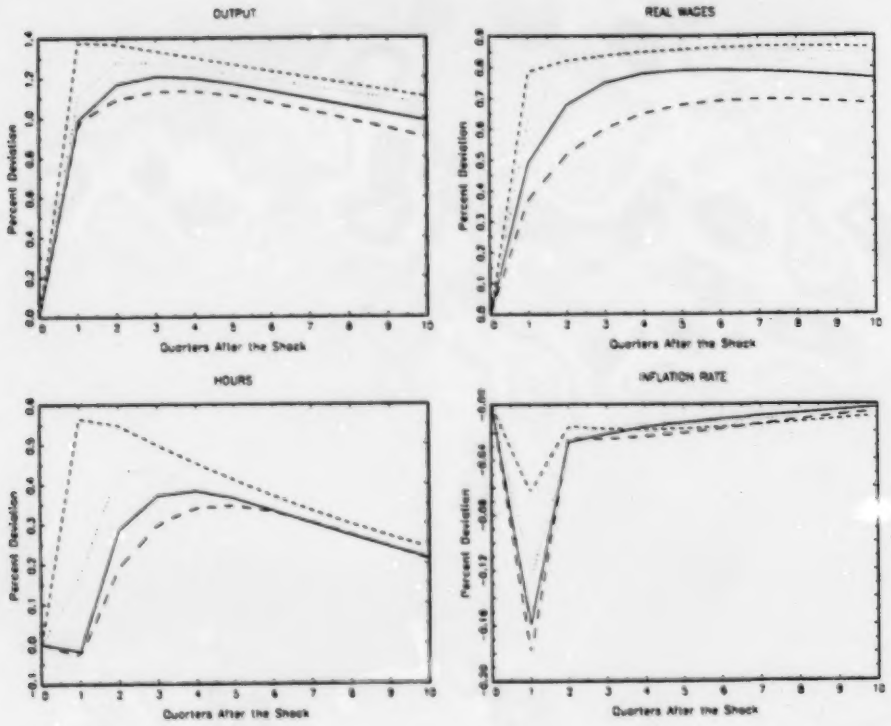
The effects of money supply shocks in the SSP model with larger ϕ_p



The impulse responses are computed for the SSP model with $\phi_p = 26.63$ (dashed line) and the SSP model with $\phi_p = 2.80$ (solid line).

Figure 3:

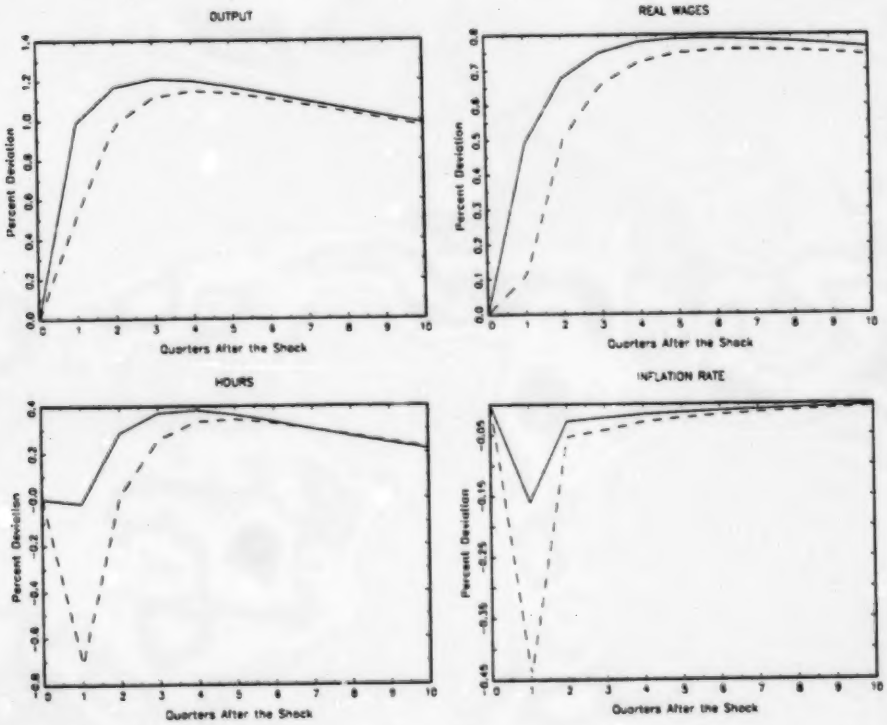
The effects of technology shocks in the four estimated models.



The impulse responses are computed for the SSP model (dashed line), the PCR model (dotted line), the PER model (large dashed line), and the PCER model (solid line).

Figure 4:

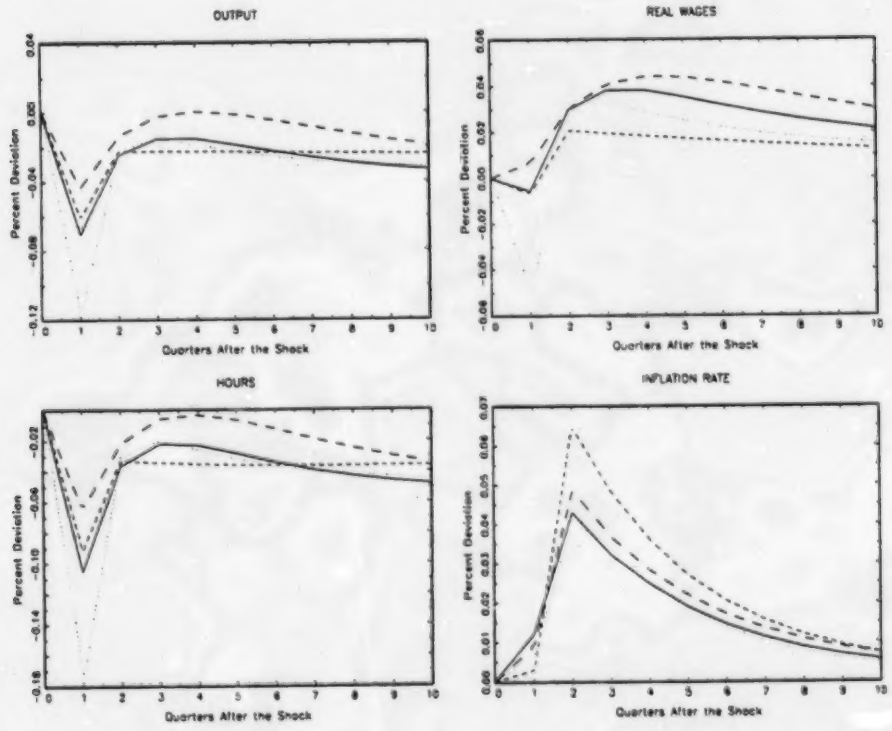
The effects of technology shocks in the estimated and exogenous PCER models



The impulse responses are computed for the estimated PCER model (solid line) and the exogenous PCER model (dashed line).

Figure 5:

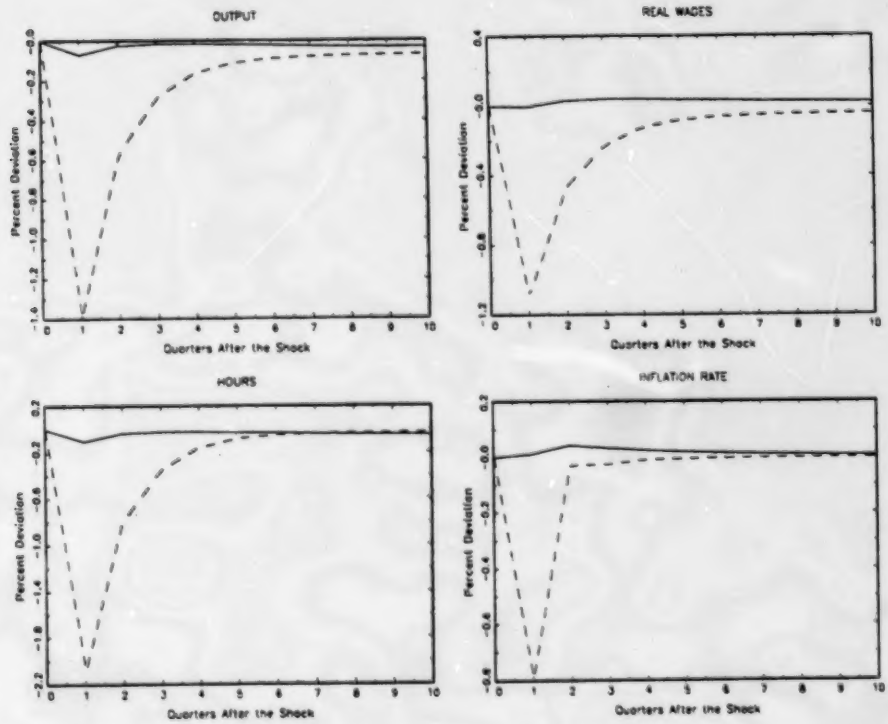
The effects of money demand shocks in the four estimated models



The impulse responses are computed for the SSP model (dashed line), the PCR model (dotted line), the PER model (large dashed line), and the PCER model (solid line).

Figure 6:

The effects of money demand shocks in the estimated and exogenous PCER Models



The impulse responses are computed for the estimated PCER model (solid line) and the exogenous PCER model (dashed line).

Appendix A: The Monopolistic Competition Equilibrium

$$\frac{c_t^{-\frac{1}{\gamma}}}{c_t^{\frac{\gamma-1}{\gamma}} + b_t^{\frac{1}{\gamma}} m_t^{\frac{\gamma-1}{\gamma}}} = \lambda_t; \tag{33}$$

$$\frac{b_t^{\frac{1}{\gamma}} m_t^{-\frac{1}{\gamma}}}{c_t^{\frac{\gamma-1}{\gamma}} + b_t^{\frac{1}{\gamma}} m_t^{\frac{\gamma-1}{\gamma}}} = \lambda_t - \beta E_t \left(\frac{\lambda_{t+1}}{\pi_{t+1}} \right); \tag{34}$$

$$\frac{\eta}{1-h_t} = \lambda_t w_t; \tag{35}$$

$$\beta E_t \left[\frac{\lambda_{t+1}}{\lambda_t} \left(r_{t+1} + \frac{\phi_k}{2} \left(\frac{i_{t+1}}{k_{t+1}} \right)^2 + (1-\delta) \left(1 + \phi_k \frac{i_{t+1}}{k_{t+1}} \right) \right) \right]$$
$$= 1 + \phi_k \left(\frac{i_t}{k_t} \right) \tag{36}$$

$$k_{t+1} = (1-\delta) k_t + i_t; \tag{37}$$

$$y_t = A_t k_t^{\alpha} (g^t h_t^{1-\alpha}); \tag{38}$$

$$\frac{\alpha y_t}{k_t q_t} = r_t; \tag{39}$$

$$\frac{(1-\alpha) y_t}{h_t q_t} = w_t + \phi_h \left(\frac{h_t}{h_{t-1}} - 1 \right) \frac{y_t}{h_{t-1}}$$
$$- \beta \phi_h E_t \left[\left(\frac{h_{t+1}}{h_t} - 1 \right) \frac{h_{t+1} y_{t+1}}{h_t^2} \frac{\lambda_{t+1}}{\lambda_t} \right]; \tag{40}$$

$$q_t^{-1} = \frac{\theta - 1}{\theta} + \frac{\phi_p}{\theta} (\pi_t - 1) \pi_t - \frac{\beta \phi_p}{\theta} E_t \left[(\pi_{t+1} - 1) \frac{\pi_{t+1} y_{t+1} \lambda_{t+1}}{y_t \lambda_t} \right]; \tag{41}$$

$$y_t = c_t + i_t + \frac{\phi_k}{2} \frac{i_t^2}{k_t} + \frac{\phi_p}{2} (\pi_t - 1)^2 y_t + \frac{\phi_h}{2} \left(\frac{h_t}{h_{t-1}} - 1 \right)^2 y_t; \tag{42}$$

$$\mu_t = \frac{m_t \pi_t}{m_{t-1}}; \tag{43}$$

$$\log(A_t) = (1-\rho_A) \log(A) + \rho_A \log(A_{t-1}) + \varepsilon_{At}; \tag{44}$$

$$\log(b_t) = (1-\rho_b) \log(b) + \rho_b \log(b_{t-1}) + \varepsilon_{bt}; \tag{45}$$

$$\log(\mu_t) = (1-\rho_\mu) \log(\mu) + \rho_\mu \log(\mu_{t-1}) + \omega_A \varepsilon_{At} + \omega_b \varepsilon_{bt} + \varepsilon_{\mu t}. \tag{46}$$

Appendix B: The Transformed Equilibrium System

$$\frac{\tilde{c}_t^{-\frac{1}{\gamma}}}{\tilde{c}_t^{\frac{\gamma-1}{\gamma}} + b_t^{\frac{1}{\gamma}} \tilde{m}_t^{\frac{\gamma-1}{\gamma}}} = \tilde{\lambda}_t; \tag{47}$$

$$\frac{b_t^{\frac{1}{\gamma}} \tilde{m}_t^{-\frac{1}{\gamma}}}{\tilde{c}_t^{\frac{\gamma-1}{\gamma}} + b_t^{\frac{1}{\gamma}} \tilde{m}_t^{\frac{\gamma-1}{\gamma}}} = \tilde{\lambda}_t - \beta E_t \left(\frac{\tilde{\lambda}_{t+1}}{g\pi_{t+1}} \right); \tag{48}$$

$$\frac{\eta}{1-h_t} = \tilde{\lambda}_t \tilde{w}_t; \tag{49}$$

$$E_t \left[\frac{\tilde{\lambda}_{t+1}}{\tilde{\lambda}_t} \left(r_{t+1} + \frac{\phi_k}{2} \left(\frac{\tilde{i}_{t+1}}{\tilde{k}_{t+1}} \right)^2 + (1-\delta) \left(1 + \phi_k \frac{\tilde{i}_{t+1}}{\tilde{k}_{t+1}} \right) \right) \right]$$
$$= \frac{g}{\beta} \left(1 + \phi_k \frac{\tilde{i}_t}{\tilde{k}_t} \right); \tag{50}$$

$$g\tilde{k}_{t+1} = (1-\delta)\tilde{k}_t + \tilde{i}_t; \tag{51}$$

$$\tilde{y}_t = A_t \tilde{k}_t^{\alpha} h_t^{1-\alpha}; \tag{52}$$

$$\frac{\alpha \tilde{y}_t}{\tilde{k}_t q_t} = r_t; \tag{53}$$

$$\frac{(1-\alpha)\tilde{y}_t}{q_t} = \tilde{w}_t h_t + \phi_h \left(\frac{h_t}{h_{t-1}} - 1 \right) \frac{h_t \tilde{y}_t}{h_{t-1}}$$
$$-\beta \phi_h E_t \left[\left(\frac{h_{t+1}}{h_t} - 1 \right) \frac{h_{t+1} \tilde{y}_{t+1}}{h_t} \frac{\tilde{\lambda}_{t+1}}{\tilde{\lambda}_t} \right]; \tag{54}$$

$$q_t^{-1} = \frac{\theta - 1}{\theta} + \frac{\phi_p}{\theta} (\pi_t - 1) \pi_t - \frac{\beta \phi_p}{\theta} E_t \left[(\pi_{t+1} - 1) \frac{\pi_{t+1} \tilde{y}_{t+1} \tilde{\lambda}_{t+1}}{\tilde{y}_t \tilde{\lambda}_t} \right]; \tag{55}$$

$$\tilde{y}_t = \tilde{c}_t + \tilde{i}_t + \frac{\phi_k}{2} \frac{\tilde{i}_t^2}{\tilde{k}_t} + \frac{\phi_p}{2} (\pi_t - 1)^2 \tilde{y}_t + \frac{\phi_h}{2} \left(\frac{h_t}{h_{t-1}} - 1 \right)^2 \tilde{y}_t; \tag{56}$$

$$\mu_t = \frac{g \tilde{m}_t \pi_t}{\tilde{m}_{t-1}}; \tag{57}$$

$$\log(A_t) = (1-\rho_A) \log(A) + \rho_A \log(A_{t-1}) + \varepsilon_{At}; \tag{58}$$

$$\log(b_t) = (1-\rho_b) \log(b) + \rho_b \log(b_{t-1}) + \varepsilon_{bt}; \tag{59}$$

$$\log(\mu_t) = (1-\rho_\mu) \log(\mu) + \rho_\mu \log(\mu_{t-1}) + \omega_A \varepsilon_{At} + \omega_b \varepsilon_{bt} + \varepsilon_{\mu t}. \tag{60}$$

Appendix C: The Steady-State Ratios

$$\mu = g\pi; \tag{61}$$

$$\frac{\tilde{c}}{\tilde{m}} = \left(\frac{\mu-\beta}{b^{\frac{1}{\gamma}}\mu}\right)^{\gamma}; \tag{62}$$

$$\tilde{\lambda}\tilde{c} = \left[1+b^{\frac{1}{\gamma}}\left(\frac{\mu-\beta}{b^{\frac{1}{\gamma}}\mu}\right)^{1-\gamma}\right]^{-1}; \tag{63}$$

$$q = \theta\left[(\theta-1)+\phi_p\pi\left(1-\beta\right)\left(\pi-1\right)\right]^{-1}; \tag{64}$$

$$\frac{\tilde{i}}{\tilde{k}} = \frac{g+\delta-1}{g}; \tag{65}$$

$$r = \frac{g}{\beta}\left(1+\phi_k\frac{\tilde{i}}{\tilde{k}}\right)-\frac{\phi_k}{2}\left(\frac{\tilde{i}}{\tilde{k}}\right)^2-(1-\delta)\left(1+\phi_k\frac{\tilde{i}}{\tilde{k}}\right); \tag{66}$$

$$\frac{\tilde{k}}{\tilde{y}} = \frac{\alpha}{rq}; \tag{67}$$

$$\frac{\tilde{c}}{\tilde{y}} = 1-\frac{\phi_p}{2}\left(\pi-1\right)^2-\frac{\tilde{i}}{\tilde{k}}\frac{\tilde{k}}{\tilde{y}}\left(1+\frac{\phi_k}{2}\frac{\tilde{i}}{\tilde{k}}\right); \tag{68}$$

$$\tilde{\lambda}h = \frac{(\tilde{\lambda}\tilde{c})}{(\tilde{c}/\tilde{y})}\left[\frac{1}{A}\left(\frac{qr}{\alpha}\right)^{\alpha}\right]^{\frac{1}{1-\alpha}}; \tag{69}$$

$$\tilde{w}\tilde{\lambda}h = \frac{(1-\alpha)}{q}\frac{(\tilde{\lambda}\tilde{c})}{(\tilde{c}/\tilde{y})}; \tag{70}$$

$$h = \frac{(\tilde{w}\tilde{\lambda}h)}{\eta+(\tilde{w}\tilde{\lambda}h)}. \tag{71}$$

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